The effect of residual stresses on bimaterial structure with initial crack located near interface
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Abstract. Residual stresses inherent to many structural components are mainly induced during fabrication. From the point of view of engineering practice, of particular interest is investigation of the heterogeneous material structures where the effects of residual stresses often can be significant. The paper considers the effect of residual stresses in bimaterial structure with initial crack located near a sharp interface. Standard compact tension (CT) specimen was considered. The influence of coefficient of thermal expansion, CTE and mechanical properties were considered in linear-elastic bimaterials. The residual stresses were induced by cooling the specimen from an elevated temperature to room temperature. Subsequently, the specimen was statically loaded. The materials on the interface were assumed homogeneous, thus there was only a jump of the material properties at a single interface. The residual stresses field produced an effect on the crack tip by inducing an additional crack driving force term, the material inhomogeneity term, $C_{inh}$. The material inhomogeneity term was evaluated by a post-processing procedure, following the conventional finite element stress analysis. The values of the J-integrals around the crack tip, $J_{tip}$, and around the external boundaries, $J_{far}$, were also computed. During the loading sequence, additionally, the experimental J-integral, $J_0$, was evaluated, according to the standard procedures from the area below the curve load, $F$, vs. load line displacement, $v_{LL}$. Based on obtained results, one can conclude that the effect of thermally induced residual stresses on the crack driving force is noticeable, even in cases of bimaterials with homogeneous mechanical properties and inhomogeneity only in CTE.

Introduction

Residual stresses are present in various structural components, because of manufacturing process, including cooling from elevated temperature. From the point of view of engineering practice, the analysis of heterogeneous material structures where the effects of residual stresses can be significant is necessary. The effect of inhomogeneity is important for understanding of the fracture behaviour of composite materials, functionally graded materials, welded and soldered joints, etc. The materials with special surface treatments, such as hardened steels, thin and thick coatings are also typical examples where sharp or graded interface between two materials exists.

Recent experimental research has shown that even small differences in CTE can significantly affect the structures with initial crack [1-3]. Moreover, it has been demonstrated that compressive residual stresses in a material of slightly lower CTE value lead even to termination of the fatigue crack growth in tested bimaterial structure for the lower values of $\Delta K$ [1].
Quantitative description of the effect of the residual stresses is essential for understanding the behaviour of inhomogeneous materials or components during static or cyclic loading. It is known long since that inhomogeneous material property can affect the effective crack driving force [4,5]. In recent papers, the concept of material forces [5,6] was used to investigate the inhomogeneity effect. It has been shown [7,8] that material inhomogeneities in the direction of the crack extension induce an additional crack driving force term, referred to as the material inhomogeneity term, \(C_{\text{inh}}\).

The effective crack driving force, stated in terms of the J-integral around the crack tip, \(J_{\text{tip}}\), is given by the sum of the far-field J-integral, \(J_{\text{far}}\), and the material inhomogeneity term,

\[
J_{\text{tip}} = J_{\text{far}} + C_{\text{inh}}
\]  

The value of the material inhomogeneity term, \(C_{\text{inh}}\), can be computed by a post-processing procedure, following a conventional numerical stress analysis obtained by finite element (FE) method. In preceding investigations, the effect of material inhomogeneities has been explored for linear-elastic and elastic-plastic bimaterial specimens with sharp and graded interfaces.

It has been found that the material inhomogeneity term, \(C_{\text{inh}}\), is positive and the effective crack driving force, \(J_{\text{tip}}\), becomes larger than the far-field J-integral, \(J_{\text{far}}\), if a crack grows towards a more compliant and/or lower-strength material. A transition towards stiffer and/or higher-strength material leads to a negative \(C_{\text{inh}}\), and \(J_{\text{tip}}\) becomes smaller than \(J_{\text{far}}\). When the material inhomogeneity term is negative, shielding effect exists in front of the crack tip, and vice versa, for positive \(C_{\text{inh}}\), anti-shielding effect occurs.

In above-mentioned researches, the effects of residual stresses have been neglected. The aim of this paper is to establish the effects of the stresses induced during the manufacturing of a component caused by cooling from an elevated temperature. For both material scientists and engineers, it is important to predict whether this effect is significant, or it can be neglected.

**Experimental**

Numerical analysis was carried out on the model of pre-cracked CT specimen made of bimaterial with a sharp interface, firstly exposed to cooling and then loaded mechanically. The model was made of two homogeneous materials, perfectly bonded along a sharp interface. The specimen width is \(W=50\) mm, thickness is \(B=25\) mm and initial crack length is \(a_0=29\) mm (see Fig. 1). Sharp material interface was perpendicular to the crack plane, and distance between the crack tip and the interface, \(L\), was varied (four values were considered: 2.5 mm, 1.25 mm, 0.3 mm and 0.15 mm).

Two values of Young's modulus were used: \(E=210\) GPa (corresponding to steel) and \(E=70\) GPa (corresponding to aluminum alloy). The Poisson's ratio was assumed constant (\(\nu=0.3\)). The materials were assumed to have different CTE; two values of CTE were considered: \(12 \times 10^{-6}\) K\(^{-1}\) (corresponding to ferritic steel), and \(24 \times 10^{-6}\) K\(^{-1}\) (corresponding to aluminum alloy).

Stress analysis was carried out using software package ABAQUS (www.hks.com). The residual stresses were introduced by cooling the specimen from an elevated temperature. Heat transfer was not considered during the process of cooling, and the introduced temperature intervals were used only to cause residual stress fields. After the cooling, the specimen was loaded statically, by prescribing the load line displacement (upper left corner in the Fig. 1). In order to reduce the computation time, two-dimensional FE model was analysed.

The FE mesh consisted of isoparametric 8-node plane stress elements. As in this paper a stationary crack was considered, singular elements were used at the crack tip. In order to attain sufficient accuracy, the FE mesh must be fine enough in the crack tip region and along the whole interface. Far-field and near-crack tip J-integrals were computed using the virtual crack extension method of ABAQUS [9].
Fig. 1: FE model of the bimaterial CT specimen with paths for evaluating the J-integrals

The material inhomogeneity term, $C_{inh}$, was evaluated using a post-processing procedure [10]:

$$C_{inh} = -\varepsilon_k \int_{\Sigma} \frac{\partial \phi (e_{pq} x_p x_q)}{\partial x_j} dA - \sum_{i=1}^{k} \int_{\Sigma^i} (\nabla \phi) - (\sigma_{ij}) (\varepsilon_{ij}) (n_j) ds$$

(2)

where $\phi$ is stored energy density that depends on linear strain, $\varepsilon$, dependent explicitly on the reference coordinate – $x$. The region $D$ denotes the area between two contours that are used to evaluate the near-tip and far-field J-integrals. It is assumed that there are $i=1,2,..,k$ sharp interfaces, $\Sigma^i$, within the region $D$. $\nabla \phi$ denotes the jump of the strain energy density and $\nabla \varepsilon_{ij}$ is the jump of the strain components at an interface; $(\sigma_{ij})$ denotes the mean value of the local stress components on both sides of the interface; $n_j$ is the unit normal vector to the interface and $e_j$ is the unit vector in the direction of the crack growth.

If both materials are homogeneous, the area integral in Eqn. (2) does not exist, thus for bimaterial with sharp interface analysed in this paper, only the second part of the equation was considered in the post-processing procedure for determination of $C_{inh}$, for details see [11].

Results and discussion

The effect of cooling temperature range of bimaterial specimens consisting of two linear-elastic materials with an inhomogeneity only in CTE, but otherwise of homogeneous mechanical properties was considered. In Fig. 2 the variation of $C_{inh}$ vs. distance $L$, for bimaterial with $E = 210$ GPa and CTE ratio = 2, for three temperature ranges $\Delta T$ is presented. From this figure, one can see that $C_{inh}$ increases with reduction of $L$ and increase of temperature range. These values, obtained during the cooling of the model, were used as initial data for the mechanical loading procedure. However, these values are rather small, except in the case where crack tip is very close to the interface.

During the mechanical loading, the influence of Young’s modulus, CTE, and distance between the crack tip and the interface was analysed. Increasing of the external loading leads to the increase of near-tip and far-field crack driving forces. Due to non-uniform distribution of residual stresses, the above-mentioned crack driving forces will not grow with same intensity, although the materials have the same elastic modulus, $E$. 
In Fig. 3 the variation of the crack driving forces during the mechanical loading (depending on $v_{LL}$) is plotted: near-tip, $J_{tip}$, and far-field, $J_{far}$, $J$-integrals, material inhomogeneity term, $C_{inh}$ and experimental value of $J$-integral, $J_0$, determined according to [12]. The case where CTE ratio = 2 ($24 \times 10^{-6}$ K$^{-1}$/12 $\times 10^{-6}$ K$^{-1}$) is considered. The conclusion is that far-field parameter of the crack driving force does not provide an actual representation of material resistance to the onset of the crack growth in case of linear-elastic bimaterial containing residual stresses, in spite of the homogeneity in Young’s modulus and Poisson's ratio. $J_{tip} > J_{far}$, which means that $C_{inh}$ is larger than zero (see Eqn. 1) and antishielding effect is exhibited.

Fig. 3: Crack driving forces $J_{tip}$, $J_{far}$, $J_0$ and $C_{inh}$ vs. load line displacement $v_{LL}$ during the mechanical loading

In further analysis, mechanical loading of the CT bimaterial specimen was considered for the cases where coupled inhomogeneity in CTE and $E$ exists. As in previous analysis, before the mechanical loading, the specimens were subjected to the cooling and the residual stresses were induced. The influence of Young’s modulus is shown in Fig. 4, for constant CTE ratio = 2. One can see that decreasing of the ratio $E_1/E_2$ decreases $C_{inh}$, thus improving the fracture resistance of the bimaterial structure.

In Fig. 5, the variation of $C_{inh}$ vs. far-field $J$-integral, $J_{far}$, for two cases of inhomogeneity in Young's modulus, $E$, is shown: stiff/compliant transition case (210/70 GPa) and compliant/stiff transition case (70/210 GPa). Two CTE ratios are considered: 2 and 0.5. On the same diagram,
computed values of the material inhomogeneity term, $C_{inh}$, for the cases of bimaterials without residual stresses are shown. Bimaterial specimen without residual stresses was not subjected to the cooling, but only to the mechanical loading, the increase of which varies the value of $C_{inh}$ due to inhomogeneity in Young's modulus.

It is noticeable that the CTE ratio larger than 1 (24 x 10^{-6} K^{-1} / 12 x 10^{-6} K^{-1}) induces unfavourable effects: for the same value of $J_{far}$, the material inhomogeneity term has higher values in comparison to the case without residual stresses. Thus, for stiff/compliant transition case the antishielding effect induced by inhomogeneity in $E$ is intensified by inhomogeneity in CTE, while in compliant/stiff transition case the shielding effect induced by inhomogeneity in $E$ is decreased (see dashed lines in Fig. 5). And vice versa, CTE ratio smaller than 1 (12 x 10^{-6} K^{-1} / 24 x 10^{-6} K^{-1}) induces favourable effect: for the same value of $J_{far}$, the material inhomogeneity term has lower values in comparison to the case without residual stresses (see solid lines in Fig. 5).

![Graph 4: The effect of the Young’s modulus, $E$, on value of $C_{inh}$](image4)

![Graph 5: Comparison between bimaterials with and without residual stresses for stiff/compliant and compliant/stiff transition cases](image5)

![Graph 6: $C_{inh}$ vs. $L$ for bimaterials with coupled inhomogeneity in CTE and $E$; a) CTE ratio = 2 and b) CTE ratio = 0.5](image6)

The effect of the distance between the crack tip and the interface, $L$, on $C_{inh}$, for bimaterials with coupled inhomogeneity is shown on Fig. 6. It is obvious that for small values of $L$ (less than 0.5 mm), the effect of inhomogeneity is significant. Even at a lower value of external loading (i.e. $J_{far} = 80$ kJ/m$^2$), the difference between positive values of $C_{inh}$ (antishielding effect) is very large (about 140 kJ/m$^2$, see Fig. 6).
Conclusion

The effect of thermally induced residual stresses on crack driving forces for bimaterials with initial crack located near the interface was analysed. Linear-elastic bimaterials with inhomogeneity in coefficient of thermal extension, CTE, and Young's modulus, \( E \), were considered. It has been shown that the far-field J-integral, \( J_{\text{far}} \), does not represent real resistance to fracture, if bimaterial contains residual stresses. This phenomenon is more prominent when the crack tip is closer to the interface. It was pointed out that compliant/stiff transition case and smaller CTE ratio decrease the values of \( C_{\text{inh}} \), thus improving the fracture resistance of the bimaterial structure.

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References